Minimum-cost-based capacity planning: does annualized capital expenditure always yield full cost recovery with marginal pricing?

Proofs of the statements

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Equations numbered with an asterisk (eq) correspond to the numbering used in the main document.

Electronic companion. Proofs of the statements

This electronic companion presents the proof of the results that are used in the paper.

EC.1. Full cost recovery for [OYAC] (one year with annualized investment cost) with long-run prices.

THEOREM 1 Let's consider the [OYAC] problem as defined in equations (1*) to (5*) with $k_j = 1 \; \forall \, j$ for the shake of simplicity. Then approximated margin M_{jy} as defined in (16*) is zero.

Proof

Let's write optimality conditions for [OYAC]. The lagrangian function would be:

$$\Omega = \{x_{jY}\,, q_{jYl,}\, z_{Yl}\}$$

$$\begin{aligned}\mathcal{L}_\Omega \; &= \sum_j \beta_{jY}\, x_{jY} + \sum_{jl} \delta_{jY}\, t_l\, q_{jYl} + \sum_l \delta_Y^{NS} t_l\, z_{Yl} \; - \sum_{jl} \rho_{jYl} q_{jYl} \; - \sum_l \rho_{Yl}^{NS} z_{Yl} \\ &\quad + \sum_{jl} \mu_{jYl}(q_{jYl} - x_{jY}) + \sum_l \pi_{Yl}\left(D_{Yl} - z_{Yl} - \sum_j q_{jYl}\right)\end{aligned} \tag{55}$$

Optimality conditions read as:

$$\frac{\partial \mathcal{L}_\Omega}{\partial x_{jY}} = \beta_{jY} - \sum_l \mu_{jYl} = 0 \qquad \forall \, j \tag{56}$$

$$\frac{\partial \mathcal{L}_\Omega}{\partial q_{jYl}} = \delta_{jY}\, t_l \; - \rho_{jYl} + \mu_{jYl} \; - \pi_{Yl} = 0 \qquad \forall \, j, l \tag{57}$$

$$\frac{\partial \mathcal{L}_{\Omega}}{\partial z_{Yl}} = \delta_{Y}^{NS} t_l - \rho_{Yl}^{NS} - \pi_{Yl} = 0 \qquad \forall\, l \tag{58}$$

Obtaining μ_{jYl} from equation (57):

$$\mu_{jYl} = \pi_{Yl} - \delta_{jY}\, t_l + \rho_{jYl} \qquad \forall j, l \tag{59}$$

Substituting in (*56*), we obtain:

$$\beta_{jY} - \sum_{l} \left(\pi_{Yl} - \delta_{jY}\, t_l + \rho_{jYl}\right) = 0 \qquad \forall j \tag{60}$$

The term in brackets is the dual variable μ_{jYl} and thus, it is only different from zero when $q_{jYl} = x_{jY}$, i.e., for those blocks in which technology *j* is producing its maximum value. Let us call $\bar{L}(j)$ the set that includes these blocks, i.e., $\bar{L}\,(j) = \{l \mid q_{jYl} = x_{jY}\}$

Assuming that $x_{jY} > 0\ \forall j$ (we are not in a trivial situation with no investment for some technologies), then in these blocks $q_{jYl} > 0$, and then $\rho_{jYl} = 0$. Consequently, (60) may be expressed as:

$$\beta_{jY} - \sum_{l \in \bar{L}(j)} \left(\pi_{Yl} - \delta_{jY}\, t_l\right) = 0 \tag{61}$$

Let us now compute approximated margin (it is called approximated because it is calculated using long-run instead of short-run market prices):

$$M_{jY} = \sum_{l} \left[q_{jYl}\, t_l \left(\frac{\pi_{Yl}}{t_l} - \delta_{jY}\right)\right] - \beta_{jY} x_{jY} \tag{62}$$

Depending on the production of each technology, load blocks will be divided into three groups:

a) The technology *j* is not producing in year *Y* and load block *l*, then $q_{jYl} = 0$, and the term in square brackets is zero.

b) The technology *j* is producing below its maximum ($0 < q_{jYl} < x_{jY}$). Thus, *j* is marginal in the load block *l* and $\left(\frac{\pi_{Yl}}{t_l} - \delta_{jY}\right)$=0. Thus, the term in square brackets is zero, and there is nothing to add.

c) The technology *j* is producing at its maximum ($q_{jYl} = x_{jYl}$). We only have to make the sum for this set of load blocks, defined as $\bar{L}(j)$:

$$M_{jY} = \sum_{l \in \bar{L}(j)} \left[t_l\, x_{jY} \left(\frac{\pi_{Yl}}{t_l} - \delta_{jY}\right)\right] - \beta_{jY}\, x_{jY} \tag{63}$$

Operating (63) yields:

$$M_{jY} = x_{jY}\left[\sum_{l\in\bar{L}(j)} \left(\pi_{Yl} - t_l\delta_{jY}\right) - \beta_{jY}\right] \quad (64)$$

Following (61), the term in square brackets is zero, so finally, we obtain:

$$M_{jY} = 0 \quad (65)$$

q.e.d.

COROLLARY to THEOREM 1. Expression (64) can also be used to compute M_{jY}^S, substituting π_{Yl} by π_{Yl}^S.

$$M_{jY}^S = x_{jY}\left[\sum_{l\in\bar{L}(j)} \left(\pi_{Yl}^S - t_l\delta_{jY}\right) - \beta_{jY}\right] \quad (66)$$

Proof

It is similar to the THEOREM 1 proof, using the corresponding [OYFC] optimality conditions.

EC.2. Cost recovery for [OYFC] (one year with fixed costs) with short-run prices.

LEMMA 1. Let $\hat{\jmath}$ (short for $\hat{\jmath}(l)$) be the most expensive (production cost) technology that is producing in block *l*. Assuming that there are no two technologies with the same production cost ($\delta_{jY} \neq \delta_{j'Y} \forall j \neq j'$), if $\hat{\jmath}$ is producing under its maximum, then $\pi_{Yl} = \pi_{Yl}^S = \delta_{\hat{\jmath}\,Y}\, t_l$.

Proof

From the [OYFC] optimality conditions:

$$\Omega = \left\{q_{jYl,}\, z_{Yl}\right\}$$

$$\frac{\partial \mathcal{L}_\Omega}{\partial q_{jYl}} = \delta_{jY}\, t_l - \rho_{jYl}^S + \mu_{jYl}^S - \pi_{Yl}^S = 0 \qquad \forall j, l \quad (67)$$

We are assuming that there are no two technologies with the same production cost ($\delta_{jY} \neq \delta_{j'Y} \forall j \neq j'$). If $\hat{\jmath}$ is producing under its maximum ($0 < q_{\hat{\jmath}Yl} < x_{\hat{\jmath}Y}$), then $\rho_{\hat{\jmath}Yl}^S = 0$; $\mu_{\hat{\jmath}Yl}^S = 0$. Thus, from (67):

$$\frac{\pi_{Yl}^S}{t_l} = \delta_{\hat{\jmath}\,Y} \quad (68)$$

From (57), we also obtain similar equations and the final result:

$$\delta_{\hat{\jmath}Y}\, t_l - \rho_{\hat{\jmath}Yl} + \mu_{\hat{\jmath}Yl} - \pi_{Yl} = 0 \quad \Big] \quad (69)$$

$$\rho_{\hat{j}Yl} = 0\ ;\ \mu_{\hat{j}Yl} = 0 \qquad\qquad \frac{\pi_{Yl}}{t_l} = \delta_{\hat{j}\,Y} \rightarrow \pi_{Yl} = \pi^S_{Yl}$$

q.e.d.

LEMMA 2. Let $\hat{j}$ (short for $\hat{j}(l)$) be the most expensive (production cost) technology that is producing in block *l*. Assuming that there are no two technologies with the same production cost ($\delta_{jY} \neq \delta_{j'Y} \forall j \neq j'$), if $\hat{j}$ is producing exactly at its maximum, and there is no non-supplied power ($z_{jY} = 0$) then $\pi_{Yl} = \pi^S_{Yl} + \mu_{\hat{j}Yl}$. That is, it is not guaranteed that long- and short-run marginal costs are the same.

Proof

Since $\hat{j}$ is producing then $\rho^S_{\hat{j}Yl} = 0$. Therefore from (67) $\pi^S_{Yl} = \delta_{\hat{j}Y}\, t_l + \mu^S_{\hat{j}Yl}$. At this point, there is a duality gap and short-run marginal cost π^S_{Yl}/t_l could be set either by $\hat{j}$ or by the next technology in increasing order of unitary production cost. Nevertheless, assuming the most common market rules, $\hat{j}$ is marginal and therefore:

$$\frac{\pi^S_{Yl}}{t_l} = \delta_{\hat{j}\,\mathrm{Y}} \qquad (70)$$

From (57) $\pi_{Yl} = \delta_{\hat{j}Y}\, t_l + \mu_{\hat{j}Yl} = \pi^S_{Yl} + \mu_{\hat{j}Yl}$.

q.e.d.

LEMMA 3. Assuming that there are no two technologies with the same production cost ($\delta_{jY} \neq \delta_{j'Y} \forall j \neq j'$), if there exists non-supplied power ($z_{jY} > 0$), then $\pi_{Yl} = \pi^S_{Yl} = \delta^{NS}_Y t_l$.

Proof.

From (58) and the optimality condition $\partial \mathcal{L}_\Omega / \partial z_{jY} = 0$ of [OYFC] it can be easily obtained that:

$$\pi_{Yl} = \pi^S_{Yl} = \delta^{NS}_Y t_l \qquad (71)$$

q.e.d.

DEFINITION. Load blocks taxonomy.

Since technologies are ordered such that $\delta_{jY} > \delta_{j'Y}$ for $j > j'$ then it is immediate that if $q_{jYl} > 0 \;\Rightarrow\; q_{j'Yl} = x_{j'Y}\ \forall j' < j$ and if $\delta_{jYl} = 0 \;\Rightarrow\; \delta_{j'Yl} = 0\ \forall j' > j$. Note that in these expressions, non-supplied energy can be considered technology *J*+1, *J* being the cardinal of the set of technologies.

Taking this into account, some blocks and sets of blocks definitions will be useful for the next steps:

$\bar{L}(j)$ Set of blocks with technology *j* producing at its maximum, i.e. μ_{jYl} >0. (Set previously defined).

$l''(j)$ Block with technology *j* producing at its maximum and being the most expensive one producing. (There will be, at most, a single block).

$L'(j)$ Set of blocks with technology *j* producing at its maximum but with more expensive technologies also producing. (As it will be seen, these are blocks in which technology *j* has income greater than production cost and, thus, it is recovering part of its investment cost).

Figure 10 shows these set blocks in a representation of a load duration curve. From the definitions, and also from the figure, it is immediate that:

$$\bar{L}\,(j) = L'(j) \cup \{l''(j)\}, \qquad l''(j) \notin L'(j) \qquad \forall\, j \qquad (72)$$

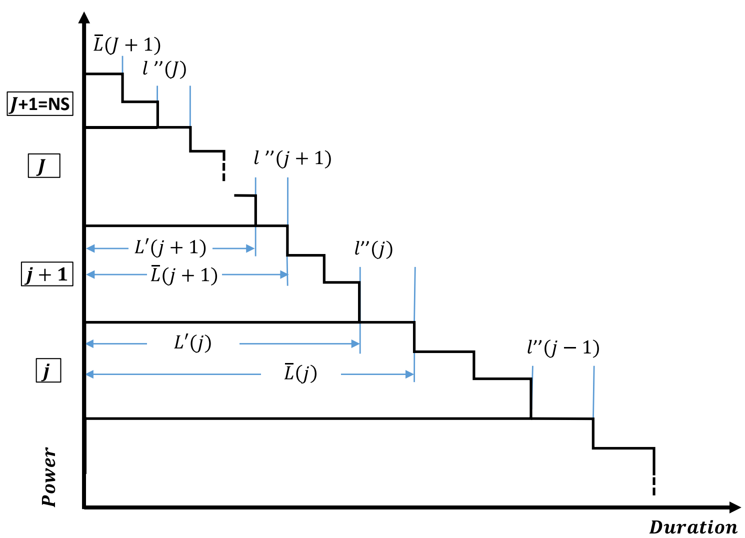

Figure 1 Load blocks sets in the load-duration curve (J is the technology with the highest variable cost and non-supplied power is represented as technology J+1).

LEMMA 4. $\mu_{jYl''(j)}$ can be expressed as a function of costs as follows:

$$\mu_{jYl''(j)} = \beta_{jY} - \beta_{j+1,Y} - \left(\delta_{j+1,Y} - \delta_{jY}\right) \sum_{l \in L'(j)} t_l \qquad (73)$$

Note: expression (73) can be extended for non-supplied power, including it as technology *J*+1, considering that $\beta_{J+1,Y} = 0$ and $\delta_{J+1,Y} = \delta_Y^{NS}$

Proof

Particularizing expression (61) for *j* and using previous set definitions:

$$\beta_{jY} = \sum_{l \in \overline{\mathrm{L}}(j)} \left(\pi_{Yl} - \delta_{jY}\, t_l\right)$$

$$= \sum_{l \in \overline{\mathrm{L}}(j+1)} \left(\pi_{Yl} - \delta_{jY}\, t_l\right) + \sum_{l \in \mathrm{L}'(j) - \overline{\mathrm{L}}(j+1)} \left(\pi_{Yl} - \delta_{jY}\, t_l\right) + \left(\pi_{Yl''(j)} - \delta_{jY}\, t_{l''(j)}\right) \quad (74)$$

Particularizing expression (61) for *j*+1 and adding a term in the second sum of the last equality that is equal to zero (using (59) for $l \in \mathrm{L}'(j) - \overline{\mathrm{L}}(j+1)$):

$$\beta_{j+1,Y} = \sum_{l \in \overline{\mathrm{L}}(j+1)} \left(\pi_{Yl} - \delta_{j+1,Y}\, t_l\right)$$

$$= \sum_{l \in \overline{\mathrm{L}}(j+1)} \left(\pi_{Yl} - \delta_{j+1,Y}\, t_l\right) + \sum_{l \in \mathrm{L}'(j) - \overline{\mathrm{L}}(j+1)} \left(\pi_{Yl} - \delta_{j+1,Y}\, t_l\right) \quad (75)$$

Substracting and using (59) for $l''(j)$, $\mu_{jYl''(j)} = \pi_{Yl''(j)} - t_{l''(j)}\delta_{jY}$

$$\beta_{jY} - \beta_{j+1,Y} = \left(\delta_{j+1,Y} - \delta_{jY}\right) \sum_{l \in \mathrm{L}'(j)} t_l + \mu_{jYl''(j)} \quad (76)$$

q.e.d.

DEFINITION. For the [OYAC] model, we will say that **the system is perfectly adapted** for a technology *j* if $\mu_{jYl''(j)} = 0$.

THEOREM 2 If the system is perfectly adapted for a technology *j*, short- and long-run prices (marginal costs), obtained from the [OYAC] and [OYFC] models, respectively, are equal, i.e. $\pi_{Yl}/t_l = \pi^S_{Yl}/t_l$, and therefore $M_{jY} = M^S_{jY}$.

Proof.

Since the system is perfectly adapted for technology *j*, then $\mu_{jYl''(j)} = 0$. Therefore, according to LEMMA 2, LEMMA 3 and LEMMA 4, the result is true for all possible situations: the technology with the highest variable cost is under its maximum or is at its maximum either with or without non-supplied demand.

q.e.d.

LEMMA 5. The margin for a technology *j* can be obtained from the margin for technology *j*+1, the immediate superior in increasing order of unitary production costs, as follows:

$$M_{jY}^{S} = x_{jY}\left[\frac{M_{j+1,Y}^{S}}{x_{j+1,Y}} - \mu_{jYl''(j)}\right] \tag{77}$$

Proof

We start from expression (66) applied to technology *j*+1. We can extend the sum in $\bar{L}(j+1)$ to $L'(j)$ by adding some null terms in the summation (taking into account (68) and that, by definition, the blocks in $L'(j) - \bar{L}(j+1)$ are blocks with production under the technology maximum).

$$\begin{aligned} M_{j+1,Y}^{S} &= x_{j+1,Y}\left[\sum_{l\in\bar{L}(j+1)}\left(\pi_{Yl}^{S} - t_l\delta_{j+1,Y}\right) - \beta_{j+1,Y}\right] \\ &= x_{j+1,Y}\left[\sum_{l\,\in\,\mathrm{L}'(j)}\left(\pi_{Yl}^{S} - t_l\delta_{j+1,Y}\right) - \beta_{j+1,Y}\right] \end{aligned} \tag{78}$$

Now we use (73) to substitute $\beta_{j+1,Y}$, and we obtain:

$$\begin{aligned} M_{j+1,Y}^{S} = x_{j+1,Y}\Bigg[\sum_{l\,\in\,\mathrm{L}'(j)}\left(\pi_{Yl}^{S} - t_l\delta_{j+1,Y}\right) - \beta_{jY} + \left(\delta_{j+1,Y} - \delta_{jY}\right)\cdot\sum_{l\,\in\,\mathrm{L}'(j)} t_l \\ + \mu_{jYl''(j)}\Bigg] = x_{j+1,Y}\left[\sum_{l\,\in\,\mathrm{L}'(j)}\left(\pi_{Yl}^{S} - t_l\delta_{jY}\right) - \beta_{jY} + \mu_{jYl''(j)}\right] \end{aligned} \tag{79}$$

Now we apply (66) for technology *j* taking into account that $\bar{L}(j) = L'(j) \cup l''(j)$.

$$M_{jY}^{S} = x_{jY}\left[\sum_{l\in L'(j)}\left(\pi_{Yl}^{S} - t_l\delta_{jY}\right) + \left(\pi_{Yl''(j)}^{S} - t_{l''(j)}\delta_{jY}\right) - \beta_{jY}\right] \tag{80}$$

Considering from (70) that $\pi_{Yl''(j)}^{S} = t_{l''(j)}\delta_{jY}$ and using (79) we obtain (77) as intended.

THEOREM 3 Let's consider the [OYFC] problem as defined in equations (10*) to (14*). The margin for each technology can be computed from the capacities x_{jY} as:

$$M_{jY}^{S} = -x_{jY}\sum_{j'=j}^{J}\mu_{jYl''(j')} \tag{81}$$

Proof.

We will start by computing $M_{J+1,Y}^{S}$ using (66):

$$M_{J+1,Y}^{S} = x_{J+1,Y}\left[\sum_{l\in\bar{L}(J)}\left(\pi_{Yl}^{S} - t_l\delta_{J+1,Y}\right) - \beta_{J+1,Y}\right] \tag{82}$$

$\beta_{J+1,Y}$ is zero (no investment cost for non-supplied demand). Since by notation $\delta_{J+1,Y} = \delta_{Y}^{NS}$, LEMMA

3 leads to $M^S_{j+1,Y} = 0$. Starting from this value and using (77) recursively, we get (81).

q.e.d.

COROLLARY 1 to *Theorem 3.*

If the system is perfectly adapted for all the technologies, then the margin is zero for all of them.

Proof.

Being $\mu_{jYl''(j)} = 0\ \forall j$, from (81) we directly obtain that $M^S_{jY} = 0\ \ \forall j$.

q.e.d.

COROLLARY 2 to *Theorem 3.*

For the general case, when the system is not perfectly adapted for all technologies, long-run prices will be larger or equal than short-run ones, and market incomes will be lower or equal than overall costs, and so $M^S_{jY} \leq 0$.

Proof.

Taking into account that by the duality properties $\mu_{jYl''(j)}$ is non-negative, it is immediate from (81) and LEMMA 2.

q.e.d.

EC.3. Bound to the dual variable that defines the difference between short- and long-run prices in [OYAC].

LEMMA 6. If the system is not perfectly adapted for technology *j*, there is a value t^*_j [time] such that $t_{1j} < t^*_j < t_{2j}$ that fullfills:

$$\beta_{jY} - \beta_{j+1,Y} - (\delta_{j+1,Y} - \delta_{jY})t^*_j = 0 \qquad (83)$$

Being:

$$t_{1j} = \sum_{l \in L'(j)} t_l$$
$$t_{2j} = \sum_{l \in \mathrm{L}'(j) \cup \{l''(j)\}} t_l = \sum_{l \in \bar{L}(j)} t_l \qquad (84)$$

Proof

An outline of the proof is provided valid for every technology *j*, except for *J,* the one with the highest expensive cost. It can be easily extended if non-supply energy is considered as technology *J*+1.

From the definition of $L'(j)$ is immediate that $l\epsilon L'(j) \Rightarrow q_{j+1,Yl} > 0$. Now $\forall l\epsilon L'(j)$ with respect to the solution to the problem, we increase q_{jYl} infinitesimally (so that the set of active constraints does not vary) and decrease $q_{j+1,Yl}$ by the same infinitesimal quantity. Additionally, to obtain feasibility, we increase x_{jY} and decrease $x_{j+1,Y}$ by the same quantity. The variation of the Lagrange function *(55)* in this new point with respect to the optimal value is positive, while we are in a minimum:

$$\beta_{jY} + t_{1j}\delta_{jY} - \beta_{j+1,Y} - t_{1j}\delta_{jY} > 0 \qquad (85)$$

On the other hand, from the definition of $\bar{L}(j)$ is immediate that $j\epsilon\bar{L}(j) \Rightarrow q_{jYl} = x_{jY}$. Now $\forall l\epsilon\bar{L}(j)$, in the same conditions as previously, we increase $q_{j+1,Yl}$ and decrease q_{jYl} , and to obtain feasibility, we increase $x_{j+1,Y}$ and decrease x_{jY}. The variation of the Lagrange function due to optimality is again positive. Changing the sign to this variation:

$$\beta_{jY} + t_{2j}\delta_{jY} - \beta_{j+1,Y} - t_{2j}\delta_{jY} < 0 \qquad (86)$$

And now, it is immediate, applying Bolzano's theorem, that the lemma is true.

LEMMA 7. $\mu_{jYl''(j)}$ is bounded, which entails, according to LEMMA 2, that the difference between short-run and long-run prices is bounded.

Proof.

If the system is perfectly adapted for technology *j*, $\mu_{jYl''(j)} = 0$

Otherwise, from (73) and Lemma 6:

$$\begin{aligned}\mu_{jYl''(j)} &= \beta_{jY} - \beta_{j+1,Y} - (\delta_{j+1,Y} - \delta_{jY})t_1 \\ &= \beta_{jY} - \beta_{j+1,Y} - (\delta_{j+1,Y} - \delta_{jY})t_{1j} + (\delta_{j+1,Y} - \delta_{jY})t_j^* \\ &- (\delta_{j+1,Y} - \delta_{jY})t_j^* = (\delta_{j+1,Y} - \delta_{jY})(t_j^* - t_{1j})\end{aligned} \qquad (87)$$

Being $t_{1j} < t_j^* < t_{2j}$. Therefore from Lemma 6, we can bound the value of $\mu_{j,l''(j)}$ as follows:

$$0 < \mu_{jYl''(j)} < (\delta_{j+1,Y} - \delta_{jY})(t_{2j} - t_{1j}) = (\delta_{j+1,Y} - \delta_{jY})t_{l''(j)} \qquad (88)$$

q.e.d.

THEOREM 4

Short term margin M_{jY}^S for any technology is bounded.

Proof.

From (81) and (88):

$$0 \geq M_{jY}^{S} > -x_{jY} \sum_{j'=j}^{J} t_{l''(j')}\left(\delta_{j'+1,Y} - \delta_{j'Y}\right) \qquad (89)$$

This also means that the shorter the blocks' lengths, the lower the difference between short- and long-run margins.

EC.4. Cost recovery for [MYAC] (multiple years with annualized investment cost) with long-run prices.

In this section, cost recovery for the [MYAC] model is analyzed considering that differences between long and short-run prices are bounded and decrease with time-periods size.

THEOREM 5 For [MYAC], margin M_j as defined in (28*) is 0.

Proof

We can write the Lagrangian function for this problem:

$$\Omega = \{x_{jy}, q_{jyl}, z_{yl}\}$$

$$\begin{aligned}
\mathcal{L}_{\Omega} = & \sum_{y=1}^{y=Y-1}\left[\frac{1}{r^{y-1}}\left(\sum_{j}(\beta_j x_{jy}) + \sum_{lj}\delta_{jy}t_l q_{jyl} + \sum_{l}\delta_{jy}^{NS}t_l z_{yl}\right)\right] + \\
& + \frac{1}{r^{Y-1}}\left(\frac{r}{r-1-g}\right)\left(\sum_{j}(\beta_j x_{jY}) + \sum_{lj}\delta_{jY}t_l q_{jYl} + \sum_{l}\delta_{jy}^{NS}t_l z_{Yl}\right) \\
& - \sum_{jyl}\rho_{jyl}q_{jyl} - \sum_{yl}\rho_{yl}^{NS}z_{yl} \\
& + \sum_{jyl}\mu_{jyl}(q_{jyl} - k_j x_{jy}) + \sum_{yl}\pi_{yl}\left(D_{yl} - z_{yl} - \sum_{j} q_{jyl}\right) \\
& - \sum_{y<Y,l}\varepsilon_{jy}(x_{j,y+1} - x_{jy})
\end{aligned} \qquad (90)$$

Deriving with respect to x_{jy} and q_{jyl} and zeroing, we obtain:

$$\frac{\partial\mathcal{L}_{\Omega}}{\partial x_{jY}} = \left(\frac{r}{r-1-g}\right)\frac{\beta_j}{r^{Y-1}} - \sum_{l} k_j\mu_{jYl} - \varepsilon_{j,Y-1} = 0 \quad \forall j \qquad (91)$$

$$\frac{\partial\mathcal{L}_{\Omega}}{\partial x_{jy}} = \frac{\beta_j}{r^{y-1}} - \sum_{l} k_j\mu_{jyl} + \varepsilon_{jy} - \varepsilon_{j,y-1} = 0 \qquad \forall j, 1 < y < Y \qquad (92)$$

$$\frac{\partial \mathcal{L}_\Omega}{\partial x_{jy}} = \frac{\beta_j}{r^{y-1}} - \sum_l k_j \mu_{jyl} + \varepsilon_{jy} = 0 \quad \forall\, j, y = 1 \tag{93}$$

$$\frac{\partial \mathcal{L}}{\partial q_{jyl}} = \frac{t_l \delta_{jy}}{r^{y-1}} - \rho_{jyl} + \mu_{jyl} - \pi_{yl} = 0 \quad \forall\, j, y, l < Y \tag{94}$$

$$\frac{\partial \mathcal{L}}{\partial z_{yl}} = \frac{t_l \delta_{jy}^{NS}}{r^{y-1}} - \rho_{yl}^{NS} - \pi_{yl} = 0 \quad \forall\; y < \mathrm{Y} \tag{95}$$

$$\frac{\partial \mathcal{L}}{\partial q_{jYl}} = \left(\frac{r}{r-1-g}\right) \frac{t_l \delta_{jY}}{r^{Y-1}} - \rho_{jYl} + \mu_{jYl} - \pi_{Yl} = 0 \quad \forall\, j, y = Y, l \tag{96}$$

$$\frac{\partial \mathcal{L}}{\partial z_{Yl}} = \left(\frac{r}{r-1-g}\right) \frac{t_l \delta_{jy}^{NS}}{r^{Y-1}} - \rho_{Yl}^{NS} - \pi_{Yl} = 0 \quad \forall l \tag{97}$$

Note: non-supplied power is included in the previous expressions as technology *J*+1, considering that $\beta_{J+1} = 0$, $\delta_{J+1,y} = \delta_y^{NS}$, $\rho_{yl} = \rho_{yl}^{NS}$ and $\mu_{J+1,Yl} = 0$.

Combining (91), (92) and (93) with (94), (95), (96) and (97), defining $\bar{L}(j)$ as in EC.2 (the set of load levels in which technology *j* is producing its maximum power), and besides taking into account that $\mu_{jyl} = 0$ if the technology *j* is producing below its upper limit since $\rho_{jyl} = 0$ by assumption, we obtain:

$$\left(\frac{r}{r-1-g}\right) \frac{\beta_j}{r^{y-1}} = \varepsilon_{j,Y-1} + \sum_{l \in \bar{L}(j)} k_j \left[\pi_{yl} - \left(\frac{r}{r-1-g}\right) \frac{t_l \delta_{jy}}{r^{y-1}}\right] \quad \forall\, j, y = Y \tag{98}$$

$$\frac{\beta_j}{r^{y-1}} = \varepsilon_{j,y-1} - \varepsilon_{jy} + \sum_{l \in \bar{L}(j)} k_j \left(\pi_{yl} - \frac{t_l \delta_{jy}}{r^{y-1}}\right) \quad \forall\, j, 1 < y < Y \tag{99}$$

$$\frac{\beta_j}{r^{y-1}} = -\varepsilon_{jy} + \sum_{l \in \bar{L}(j)} k_j \left(\pi_{yl} - \frac{t_l \delta_{jy}}{r^{y-1}}\right) \quad \forall\, j, y = 1 \tag{100}$$

Now we compute M_j as defined in (28*), (27*) and (26*), for a company owning all the capacity of a single technology *j* (the result can be easily extended to a company that has a partial capacity of several technologies).

$$M_j^S \approx M_j = \sum_{y=1}^{Y-1} \frac{M_{jy}}{r^{y-1}} + \left(\frac{r}{r-1-g}\right)\frac{M_{jY}}{r^{Y-1}}$$

$$= \sum_{y=1}^{Y-1} \frac{1}{r^{y-1}} \left\{ \sum_l \left[t_l q_{jyl} \left(\frac{\pi_{yl}}{t_l} r^{y-1} - \delta_{jy} \right) \right] - \beta_j x_{jy} \right\}$$

$$+ \frac{1}{r^{Y-1}} \left(\frac{r}{r-1-g}\right) \sum_l \left\{ t_l q_{jYl} \left[\frac{\pi_{Yl}}{t_l} r^{Y-1} \left(\frac{r-1-g}{r}\right) - \delta_{jY} \right] \right.$$

$$\left. - \beta_j x_{jY} \right\} \qquad (101)$$

$$= \sum_{y=1}^{Y-1} \left\{ x_{jy} k_j \left[\sum_{l \in \bar{L}(j)} \left(\pi_{yl} - \frac{t_l \delta_{jy}}{r^{y-1}} \right) - \frac{\beta_j}{k_j r^{y-1}} \right] \right\}$$

$$+ x_{jY} k_j \left[\sum_{l \in \bar{L}(j)} \left(\pi_{Yl} - \frac{t_l \delta_{jY}}{r^{Y-1}} \left(\frac{r}{r-1-g}\right) \right) \right.$$

$$\left. - \frac{\beta_j}{k_j r^{Y-1}} \left(\frac{r}{r-1-g}\right) \right]$$

This expression, according to (98), (99) and (100), is:

$$M_j = x_{j1}\varepsilon_{j1} + \sum_{y=2}^{Y-1} \{ x_{jy} (\varepsilon_{jy} - \varepsilon_{j,y-1}) \} - x_{jY}\varepsilon_{j,Y-1} = \sum_{y=1}^{Y-1} \{ \varepsilon_{jy} (x_{j,y+1} - x_{jy}) \} = 0 \qquad (102)$$

This last expression is zero since either $\varepsilon_{jy} = 0$ or $x_{j,y+1} = x_{jy}$ due to the slackness complementary conditions.

q.e.d.

COROLLARY 1 to Theorem 5.

If monotonicity constraint (25*) is not active for two consecutive years *y*-1 and y, and for a technology *j* (no investment is made on year *y*), full cost recovery is obtained individually for year *y*. Otherwise, if it is active, extra cost recovery is obtained in year *y*-1, but it is compensated with negative cost recovery in the next year *y*.

Proof.

This is immediate from (98), (99) and (100).

COROLLARY 2 to Theorem 5.

If there is an investment in the last year for every technology, then the margin $\bar{M}$ (computed for the finite year set from 1 to *Y*) is zero.

Proof.

This is immediate from(30*), (31*), (98) and (26*).

EC.5. Cost recovery for [MYOC] (multiple year with overall investment cost) with long-run prices.

This section shows under which circumstances the value of margin for the [MYOC] model is zero, i.e., when cost recovery is achieved for investments.

THEOREM 6 From [MYOC], if we use a modified overall cost term as defined in **¡Error! No se encuentra el origen de la referencia.**:

- For investment in a technology *j* in a year $\hat{y}$ that close before the last year or in the last year of the study horizon, i.e. $\hat{y} + s(j) - 1 \leq Y$, there is always cost recovery, i.e., $M_j^O(\hat{y}) = 0$.
- For technologies *j* that close after last year, i.e. $\hat{y} + s(j) - 1 > Y$:

$$M_j^O(\hat{y}) = \mathrm{v}_{j\hat{y}}^{\mathrm{O}} \cdot \varepsilon_{jY}^{O} \cdot \left[\left(\frac{1+g}{r}\right) - \left(\frac{r-1-g}{r}\right) \sum_{y=Y+1}^{\hat{y}+s(j)-1} r^{Y-y} \right] \qquad (103)$$

$$\forall j, \forall y: Y - s(j) + 1 \leq \hat{y}$$

Proof.

This proof will be divided into three steps.

Step 1. Previous operations with optimality conditions.

We start with the Lagrangian function of [MYOC] as defined in equations (37*) to (43*).

$$\Omega = \{x_{jy}^O, v_{jy}^o, q_{jyl}^O, z_{yl}^O\} \qquad (104)$$

$$
\begin{aligned}
\mathcal{L}_\Omega = \sum_{y=1}^{Y} & \left[\frac{1}{r^{y-1}} \left(\sum_j B^*_{jy} v^O_{jy} + \sum_{jl} \delta_{jy} t_l q^O_{jyl} + \sum_l \delta^{NS}_{jy} t_l z^O_{yl} \right) \right] \\
& + \frac{1}{r^{Y-1}} \left(\frac{1+g}{r-1-g} \right) \left(\sum_j \beta_j x^O_{jY} + \sum_{jl} \delta_{jY} t_l q^O_{jYl} + \sum_l \delta^{NS}_{jY} t_l z^O_{Yl} \right) \\
& - \sum_{jyl} \rho^O_{jyl} q^O_{jyl} - \sum_{yl} \rho^{NSO}_{yl} z^O_{yl} - \sum_{jyl} \mu^O_{jyl} \left(k_j x^O_{jY} - q^O_{jyl} \right) \\
& + \sum_{yl} \pi^O_{yl} \left(D_{yl} - \sum_j q^O_{jyl} - z^O_{yl} \right) - \sum_{jy} \varepsilon^O_{jy} v^O_{jy} \\
& + \sum_{jy} \lambda^O_{jy} \left[\left(\sum_{i=max[1,y-s(j)+1]}^{y} v^O_{ji} \right) - x^O_{jy} \right]
\end{aligned}
\tag{105}
$$

Note that dual variables ρ, σ, μ and ε must be non-negative at the solution.

KKT optimality conditions are:

$$
\frac{\partial \mathcal{L}_\Omega}{\partial x^O_{jy}} = -k_j \sum_l \mu^O_{jyl} - \lambda^O_{jy} \sum_l \mu^O_{jyl} \cdot k_j - \lambda^O_{jy} = 0 \qquad \forall j, \forall y < Y \tag{106}
$$

$$
\frac{\partial \mathcal{L}_\Omega}{\partial x^O_{jY}} = \frac{1}{r^{Y-1}} \left(\frac{1+g}{d-g} \right) \beta_j - \sum_l \mu^O_{jYl} \cdot k_j - \lambda^O_{jy} = 0 \qquad \forall j \tag{107}
$$

$$
\frac{\partial \mathcal{L}_\Omega}{\partial v^O_{jy}} = \frac{1}{r^{y-1}} B^*_{jy} - \varepsilon^O_{jy} + \sum_{i=y}^{min[y+s(j)-1,Y]} \lambda^O_{jy} = 0 \qquad \forall j, y \tag{108}
$$

$$
\frac{\partial \mathcal{L}_\Omega}{\partial q^O_{jyl}} = \frac{1}{r^{y-1}} \delta_{jy} t_l - \rho^O_{jyl} + \mu^O_{jyl} - \pi^O_{yl} = 0 \qquad \forall j, \forall y < Y, \forall l \tag{109}
$$

$$
\frac{\partial \mathcal{L}_\Omega}{\partial q^O_{jYl}} = \frac{1}{r^{Y-1}} \delta_{jY} t_l \left[1 + \left(\frac{1+g}{d-g} \right) \right] - \rho^O_{jYl} + \mu^O_{jyl} - \pi^O_{Yl} = 0 \qquad \forall j, l \tag{110}
$$

$$
\frac{\partial \mathcal{L}_\Omega}{\partial z^O_{yl}} = \frac{1}{r^{y-1}} \delta^{NS}_{jy} t_l - \rho^{NSO}_{yl} - \pi^O_{yl} = 0 \qquad \forall y < Y, \forall l \tag{111}
$$

$$
\frac{\partial \mathcal{L}_\Omega}{\partial z^O_{Yl}} = \frac{1}{r^{Y-1}} \delta^{NS}_{jY} t_l \left[1 + \left(\frac{1+g}{d-g} \right) \right] - \rho^{NSO}_{Yl} - \pi^O_{Yl} = 0 \qquad \forall \ l \tag{112}
$$

The sum index in equation (108) shows that the investment in a particular year y is active during $s(j)$ years (or less if last year Y is reached). For example, if Y=20 and $s(j)$=10, an investment made in y=4 is active from year 4 to 23, both included. If the investment year is y=14, it is active from year 14 to 20.

Step 2.

Lemma 8. The following two expressions connecting B^* (overall investment cost), δ (production costs), and β (annualized investment cost) hold (they will be useful to compute incomes and costs).

$$\frac{B_{jy}^*}{r^{y-1}} = \varepsilon_{jy}^O + \sum_{i=y}^{y+s(j)-1} k_j \cdot \sum_{l\in \bar{L}(j,y)} \left(\pi_{il}^O - \frac{\delta_{ji} t_l}{r^{i-1}}\right) = 0 \tag{113}$$

$$\forall\, j, \forall\, y < \; Y - s(j) + 1$$

$\bar{L}(j,y)$ is the set including load blocks of year *y* with technology *j* generating at maximum power.

$$\begin{aligned}\frac{B_{jy}^*}{r^{y-1}} = \varepsilon_{jy}^O + \sum_{i=y}^{Y-1} k_j \cdot \sum_{l\in \bar{L}(j,y)} \left(\pi_{il}^O - \frac{\delta_{ji} t_l}{r^{i-1}}\right) \; - \frac{1}{r^{Y-1}}\left(\frac{1+g}{r-1-g}\right)\beta_j \\ + \sum_{l\in \bar{L}(j,Y)} \left\{k_j \cdot \left[\pi_{Yl}^O - \frac{\delta_{jY} t_l}{r^{Y-1}}\left(1 + \frac{1+g}{r-1-g}\right)\right]\right\}\end{aligned} \tag{114}$$

$$\forall\, j, \forall y \colon Y - s(j) + 1 \le y$$

Proof.

We obtain μ_{jyl}^O from (109) and (110):

$$\mu_{jyl}^O = \pi_{yl}^O - \frac{\delta_{jy} t_l}{r^{y-1}} + \rho_{jyl}^O \qquad \forall\, j, \forall\, y < Y, \forall\, l \tag{115}$$

$$\mu_{jYl}^O = \pi_{Yl}^O - \frac{\delta_{jY} t_l}{r^{Y-1}}\left[1 + \left(\frac{1+g}{r-1-g}\right)\right] + \rho_{jYl}^O \qquad \forall\, j, l \tag{116}$$

Introducing (115) and (116) in (106) and (107), respectively, we obtain two expressions for $-\lambda_{jy}^O$:

$$-\lambda_{jy}^O = \sum_l k_j \cdot \left(\pi_{yl}^O - \frac{\delta_{jy} t_l}{r^{y-1}} + \rho_{jyl}^O\right) \qquad \forall\, j,\, \forall\, y < Y \tag{117}$$

$$\begin{aligned}-\lambda_{jY}^O = -\frac{1}{r^{Y-1}}\left(\frac{1+g}{r-1-g}\right)\beta_{jY} \\ + \sum_l \left\{k_j \cdot \left[\pi_{Yl}^O - \frac{\delta_{jY} t_l}{r^{Y-1}}\left(1 + \left(\frac{1+g}{r-1-g}\right)\right) + \rho_{jYl}^O\right]\right\} \qquad \forall\, j\end{aligned} \tag{118}$$

And now, we introduce (117) and (118) separately in (108). Two different expressions result depending on the value of y.

If $min[y + s(j) - 1,\; Y] < Y$ that is $y < Y - s(j) + 1$ (a plant built in year y and closed before year *Y*):

$$\frac{B_{jy}^*}{r^{y-1}} = \varepsilon_{jy}^O + k_j \sum_{i=y}^{y+s(j)-1} \sum_l \left(\pi_{il}^O - \frac{\delta_{ji} t_l}{r^{i-1}} + \rho_{jil}^O \right) = 0 \;\; \forall\, j, \forall\, y < \; Y - s(j) + 1 \qquad (119)$$

If $min[y + ls(j) - 1,\ Y] = Y$ that is $y \geq Y - ls(j) + 1$ (a plant built in year *y*, is closed in year *Y* or after):

$$\frac{B_{jy}^*}{r^{y-1}} = \varepsilon_{jy}^O + k_j \sum_{i=y}^{Y-1} \sum_l \left(\pi_{il}^O - \frac{\delta_{ji} t_l}{r^{i-1}} + \rho_{jil}^O \right) - \frac{1}{r^{Y-1}} \left(\frac{1+g}{r-1-g} \right) \beta_j$$

$$+ \sum_l \left[\pi_{Yl}^O - \frac{\delta_{jY} t_l}{r^{Y-1}} \left(1 + \left(\frac{1+g}{r-1-g} \right) \right) + \rho_{jYl}^O \right] \quad \forall\, j \qquad (120)$$

$$\forall\, j, \forall y \colon Y - s(j) + 1 \leq y$$

Let us consider the term in parenthesis in expression (119), which is equal to μ_{jil}^O according to (115). It is only different from zero in the load blocks with technology *j* producing at maximum production, i.e., $q_{jil}^O = \; k_j^O x_{ji}^O$. This load blocks also fulfil that $\rho_{jil}^O = 0$. This way it is easy to prove that according to the definition of $\bar{L}(j, y)$, we can rewrite (119) and (120) to obtain (113) and (114) as intended.

q.e.d.

Step 3. **Margin computation (using long-run prices)**

Now we will compute the margin for a particular investment v_{jy}^O and then we will try to simplify it by using the previous expressions. We will use the definition of marginal costs (prices) from(26*).

Step 3.1. Margin for an investment that closes before year *Y*.

In order to compute it, we need to determine the production of a technology *j* corresponding to an investment $v_{j\hat{y}}^O > 0$ made in a particular year $\hat{y}$, during the upcoming years. Assuming $\hat{y} < \; Y - s(j) + 1$, the margin actualized to year 1 can be computed as:

$$M_j^O(\hat{y}) = \sum_{y=\hat{y}}^{\hat{y}+s(j)-1} \left\{ \frac{1}{r^{y-1}} \sum_l \left[t_l \cdot q_{jyl}^O(\hat{y}) \cdot \left(p_{yl}^O - \delta_{jy} \right) \right] \right\} - \frac{B_{j\hat{y}}\, v_{j\hat{y}}^O}{r^{\hat{y}-1}} \;\; \forall\, j\,, \qquad (121)$$

$$\forall\, \hat{y} < \; Y - s(j) + 1$$

Depending on the load block, there may be three situations:

a) Technology *j* is not producing in year *y* and load block *l*, then $q_{yjl}^O(\hat{y}) = 0$, and the term in square brackets is zero.

b) Technology *j* produces in year *y* and load block *l* below its maximum. Thus, *j* is marginal in year *y* and load block *l*, and $\left(p_{yl}^{O}-\delta_{jy}\right)$=0, and again the term in square brackets is zero.

c) Technology *j* is producing in year *y* and load block *l* at its maximum, i.e., $q_{jyl}^{O}(\hat{y})=k_j v_{j\hat{y}}^{O}$. Note that *j* can be the most expensive technology producing or not, and so it can be setting the market price or not according to the previous short-run energy marginal cost definition formulated in (10). The summation with the term in brackets can be performed over the set $\bar{L}(j,y)$, i.e:

$$M_j^O(\hat{y})=v_{j\hat{y}}^{O}\cdot\left(k_j\sum_{y=\hat{y}}^{\hat{y}+s(j)-1}\left\{\sum_{l\in\bar{L}(j,y)}\left(\frac{p_{yl}^{O}t_l}{r^{y-1}}-\frac{\delta_{jy}t_l}{r^{y-1}}\right)\right\}-\frac{B_{j\hat{y}}}{r^{y-1}}\right) \quad (122)$$

$$\forall\, j, \forall\, \hat{y}<\; Y-s(j)+1$$

Combining this expression with (113) particularized for year $\hat{y}$ and using (44*), if $B_{j\hat{y}}^{*}\;=B_{j\hat{y}}$ then:

$$M_j^O(\hat{y})=v_{j\hat{y}}^{O}\cdot\left(-\varepsilon_{j\hat{y}}^{O}\right)\quad \forall\, j, \forall\, \hat{y}<\; Y-s(j)+1 \quad (123)$$

Considering that $\varepsilon_{j\hat{y}}^{O}$=0 because $v_{j\hat{y}}^{O}>0$, we conclude that if $B_{j\hat{y}}^{*}\;=B_{j\hat{y}}$ then $M_{j\hat{y}}^{O}=0$, and the investment recovers full costs through market incomes.

Step 3.2.a. Initial formulation of margin for an investment that closes in year *Y*.

Again, to compute the margin actualized to year 1 for an investment that closes in year *Y*, we need to determine the production of an investment $v_{j\hat{y}}^{O}>0$, made in a particular year $\hat{y}$, during the following years. Assuming now that $\hat{y}\geq\; Y-s(j)+1$, the margin actualized to year 1 can be computed as (not that the expression now is longer, taking into account years that are part of the residual value):

$$M_j^O(\hat{y})=\sum_{y=\hat{y}}^{Y}\left\{\frac{1}{r^{y-1}}\sum_{l}\left[t_l\cdot q_{jyl}^{O}(\hat{y})\cdot\left(p_{yl}^{O}-\delta_{jy}\right)\right]\right\}$$

$$+\sum_{y=Y+1}^{\hat{y}+s(j)-1}\left\{\frac{1}{r^{y-1}}\sum_{l}\left[t_l\cdot q_{jyl}^{O}(\hat{y})\cdot\left(p_{Yl}^{O}-\delta_{jY}\right)\right]\right\} \quad (124)$$

$$-\frac{B_{j\hat{y}}\,v_{j\hat{y}}^{O}}{r^{\hat{y}-1}}\quad \forall\, j\,, \forall\, \hat{y}\geq\; Y-s(j)+1$$

For example, if *Y*=20, *s*(*j*)=10 and $\hat{y}$=14, we will add 7 years in the first sum, from 14 to 20, both included. And we are adding 3 additional repetitions of the margin at year 20 in the second sum. This way, we complete 10 years, which is the life span of this technology.

There may be three situations depending on the load block, as in step 3.1. With similar reasoning, we obtain:

$$M_j^O(\hat{y}) = \sum_{y=\hat{y}}^{Y} \left\{ \frac{1}{r^{y-1}} \sum_{l \in \bar{L}(j,y)} \left[t_l \cdot k_j \cdot v_{j\hat{y}}^O \cdot \left(p_{yl}^O - \delta_{jy} \right) \right] \right\} + \sum_{y=Y+1}^{\hat{y}+s(j)-1} \left\{ \frac{1}{r^{y-1}} \sum_{l \in \bar{L}(j,y)} \left[t_l \cdot k_j \cdot v_{j\hat{y}}^O \cdot \left(p_{Yl}^O - \delta_{jY} \right) \right] \right\} - \frac{B_{j\hat{y}} \cdot v_{j\hat{y}}^O}{r^{\hat{y}-1}} \quad \forall j, \forall \hat{y} \geq Y - s(j) + 1 \tag{125}$$

Reorganizing terms:

$$M_j^O(\hat{y}) = v_{j\hat{y}}^O \left(k_j \sum_{y=\hat{y}}^{Y} \left\{ \sum_{l \in \bar{L}(j,y)} \left(\frac{p_{yl}^O t_l}{r^{y-1}} - \frac{\delta_{jy} t_l}{r^{y-1}} \right) \right\} + k_j \sum_{y=Y+1}^{\hat{y}+s(j)-1} \left\{ \sum_{l \in \bar{L}(j,y)} \left(\frac{p_{Yl}^O t_l}{r^{y-1}} - \frac{\delta_{jY} t_l}{r^{y-1}} \right) \right\} - \frac{B_{j\hat{y}}}{r^{\hat{y}-1}} \right) \quad \forall j, \forall \hat{y} \geq Y - s(j) + 1 \tag{126}$$

Step 3.2.b. Some previous operations to introduce prices in relationships that had been obtained from KKT conditions.

Before going back to the margin, we combine (114) with (44*) to cancel π_{yl}^O and π_{Yl}^O. We obtain (note that in step 3, $Y - s(j) + 1 \leq y$):

$$\frac{B_{jy}^*}{r^{y-1}} = \varepsilon_{jy}^O + k_j \cdot \sum_{i=y}^{Y-1} \sum_{l \in \bar{L}(j,y)} \left(\frac{p_{il}^O t_l}{r^{i-1}} - \frac{\delta_{ji} t_l}{r^{i-1}} \right) - \frac{1}{r^{Y-1}} \left(\frac{1+g}{r-1-g} \right) \beta_j + k_j \sum_{l \in \bar{L}(j,y)} \left[\left(\frac{p_{Yl}^O t_l}{r^{Y-1}} - \frac{\delta_{jY} t_l}{r^{Y-1}} \right) \left(1 + \left(\frac{1+g}{r-1-g} \right) \right) \right] \quad \forall j, \forall y\colon Y - s(j) + 1 \leq y \tag{127}$$

For *y=Y,* we obtain the following:

$$\frac{B_{jY}^*}{r^{Y-1}} = \varepsilon_{jY}^O - \frac{1}{r^{Y-1}} \left(\frac{1+g}{r-1-g} \right) \beta_j + \sum_{l \in \bar{L}(j,Y)} \left\{ k_j \cdot \left[\left(\frac{p_{Yl}^O t_l}{r^{Y-1}} - \frac{\delta_{jY} t_l}{r^{Y-1}} \right) \left(1 + \frac{1+g}{r-1-g} \right) \right] \right\} \quad \forall j \tag{128}$$

The definition previously made (36*) fulfills that $B^*_{jY} = \beta_j$ so we will assume this, then simplifying:

$$\beta_j = r^{Y-1}\left(\frac{r-1-g}{r}\right)\varepsilon^O_{jY} + \sum_{l\in\bar{L}(j,Y)} \{k_j[(p^O_{Yl}t_l - \delta_{jY}t_l)]\} \qquad \forall\, j \tag{129}$$

This expression is interesting because the dual variable ε^O_{jY} shows if the prices of last year (and residual years) are coupled with the previous years.

Using this expression, (127) turns out to be (note that there is a sum that has been extended from *Y*-1 to *Y* by including a new term in it):

$$\frac{B^*_{jy}}{r^{y-1}} = \varepsilon^O_{jy} + k_j \cdot \sum_{i=y}^{Y} \sum_{l\in\bar{L}(j,y)} \left(\frac{p^O_{il}t_l}{r^{i-1}} - \frac{\delta_{ji}t_l}{r^{i-1}}\right) - \varepsilon^O_{jY}\left(\frac{1+g}{r}\right) \quad \forall\, j, \forall y{:}\, Y - s(j) + 1 \le y \tag{130}$$

Step 3.2.c Final computation of margin for an investment that closes in year Y or after

Now we can go back to computing margin. If we particularize (130) for the year $\hat{y}$ ($\varepsilon^O_{j\hat{y}} = 0$ because $v^O_{j\hat{y}} > 0$) and introduce it in the first term of (126) and use again (129) for the second term. Then we have:

$$M^O_j(\hat{y}) = v^O_{j\hat{y}}\left\{\frac{B^*_{j\hat{y}}}{r^{\hat{y}-1}} + \varepsilon^O_{jY}\left(\frac{1+g}{r}\right) + \sum_{y=Y+1}^{\hat{y}+s(j)-1}\left[\frac{\beta_{jY} - r^{Y-1}\left(\frac{r-1-g}{r}\right)\varepsilon^O_{jY}}{r^{y-1}}\right] - \frac{B_{j\hat{y}}}{r^{\hat{y}-1}}\right\} \forall\, j \tag{131}$$

$$\forall\, j,\ \forall y{:}\ Y - \mathrm{s}(j) + 1 \le \hat{y}$$

Simplifying:

$$M^O_j(\hat{y}) = v^O_{j\hat{y}}\left\{\frac{B^*_{j\hat{y}}}{r^{\hat{y}-1}} + \varepsilon^O_{jY}\left(\frac{1+g}{r}\right) + \beta_{jY}\sum_{y=Y+1}^{\hat{y}+s(j)-1} r^{-y+1} - \left(\frac{r-1-g}{r}\right)\varepsilon^O_{jY}\sum_{y=Y+1}^{\hat{y}+s(j)-1} r^{Y-y} - \frac{B_{j\hat{y}}}{r^{\hat{y}-1}}\right\} \forall\, j \tag{132}$$

If we now define $B^*_{j\hat{y}}$ such that (note that this is compatible with our previous hypothesis $B^*_{jY} = \beta_j$):

$$B^*_{j\hat{y}} = B_{j\hat{y}} - \beta_j \sum_{y=Y+1}^{\hat{y}+s(j)-1} r^{\hat{y}-y} \qquad \forall\, j \tag{133}$$

We have:

$$M_j^O(\hat{y}) = v_{j\hat{y}}^O \cdot \varepsilon_{jY}^O \cdot \left[\left(\frac{1+g}{r}\right) - \left(\frac{r-1-g}{r}\right) \sum_{y=Y+1}^{y=\hat{y}+s(j)-1} r^{Y-y} \right] \quad \forall\, j, \forall y\colon Y - s(j) + 1 \le \hat{y} \tag{134}$$

q.e.d. (THEOREM 6)

COROLLARY 1 to THEOREM 6

It is immediate from (134) that for technologies *j* that close after last year, i.e. $\hat{y} + s(j) - 1 > Y$, if there is investment in the last year of the studied horizon ($v_{jY}^O > 0$), then the dual variable $\varepsilon_{jY}^O = 0$ and $M_j^O(\hat{y}) = 0$.

COROLLARY 2 to THEOREM 6

It is again immediate from the previous corollary that if for all the technologies *j* that close after last year, i.e. $\hat{y} + s(j) - 1 > Y$, there is an investment in the last year of the studied horizon, then the total margin $M^O(\hat{y})$ as defined in (47*) is zero.

COROLLARY 3 to THEOREM 6

From the previous corollaries and since $q_{jyl}^O(\hat{y}) = k_j v_{j\hat{y}}^O \ \forall\, l \in \bar{L}(j,Y)$ in equation (129), it can be deduced that for all the technologies *j* that close after last year, i.e. $\hat{y} + s(j) - 1 > Y$, if there is investment in the last year of the studied horizon, the finite total margin, as defined in (52*), is $\bar{M}^O(\hat{y}) = M^O(\hat{y}) \ = \ 0$.

EC.6. Extension. Consideration of previously existing quantity-limited production plants with null production cost.

Minimum cost capacity planning for a single year *Y* obtained from the [OYAC] model can be extended to include a preexisting quantity limited production S_Y for technologies *j* with null production cost (an example in the context of electricity productions are hydro technologies). If h_{Yl} is the production of these technologies in each block, the resulting extended model [OYAC-H] formulation is (for the sake of clarity, neither upper nor lower power limits are considered):

[OYAC-H]

$$min_{x_{jY}, q_{jYl}, h_{Yl}}\, C_Y = \sum_j (\beta_j x_{jY}) + \sum_{jl} (t_l \delta_{jY} q_{jYl}) + \sum_l (t_l \delta_Y^{NS} z_{Yl}) \tag{135}$$

s.t.

$$0 \le q_{jYl} \quad : \quad \rho_{jYl} \qquad \forall\, j, l \tag{136}$$

$$0 \leq z_{Yl} \quad : \quad \rho_{Yl}^{NS} \quad \forall\, l \tag{137}$$

$$q_{jYl} \leq k_j x_{jY} \quad : \quad \mu_{jYl} \quad \forall\, j, l \tag{138}$$

$$\sum_{j} \left(q_{jYl}\right) + h_{Yl} + z_{Yl} = D_{Yl} \quad : \quad \pi_{Yl} \quad \forall\, l \tag{139}$$

$$\sum_{l} (t_l h_{Yl}) = S_Y \quad : \quad \sigma_Y \tag{140}$$

It can be easily proved that the conclusions obtained for [OYAC] still hold for [OYAC-H]. Indeed, if we reformulate [OYAC-H] by subtracting from the demand of each load block D_{Yl} the amount h_{Yl}, we can obtain again [OYAC] model. The solution to this reformulated problem is the same and dual variables do not change since quantity-limited production has no associated cost in the objective function.

This extension could also be analogously included in the rest of the proposed models of this paper. It could also be proved similarly how the corresponding conclusions are still valid.